STABILITY OF BOUNDARY BETWEEN TWO NONISOTHERMAL MAGNETIZABLE FLUIDS

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The nature of the interconnection of the thermocapillary mechanism of convective instability and the magnetic mechanism of surface instability is investigated as a function of the heat-transfer conditions and the characteristic of the adjoining media.

The thermocapillary stability of a layer of magnetizable liquid was studied in [1, 2] for heat transfer at the free surface described by Newton's law. Above the plane upper boundary of the layer was a nonmagnetic gas of negligible density and viscosity. In this formulation the problem is considerably simplified since it is not necessary to solve the hydrodynamic and heat equations in the gas.

In practice the layer of magnetizable liquid is often bounded by a layer of liquid which does not mix with it. In such a case it is clearly necessary to consider the dynamics of both media. In this connection we investigate below the stability of a horizontal layer of nonisothermal magnetizable liquid bounded below ($z = -l$) by a plane surface separating it from a solid nonmagnetic body, and above ($z = 0$) by a plane surface separating it from an infinite mass of immiscible liquid which is also magnetizable.

The whole system is in a gravitational field directed vertically downward and a uniform magnetic field transverse to the layer. In addition, a temperature distribution with a constant vertical gradient is maintained in the layer. The z axis of a Cartesian coordinate system is directed vertically upward transverse to the layer, and the x and y axes are along the layer. We consider the interaction of the thermocapillary mechanism of convective instability with the magnetic mechanism of surface instability and neglect all other mechanisms of convective instability (gravitational and magnetic) in comparison with the thermocapillary mechanism. This is valid, in particular, for sufficiently thin layers, and is mathematically equivalent to the limit $K = 0$, $\beta = 0$.

By using the results of [1, 2] it is easy to write down the linearized thermomechanical equations for a magnetizable liquid and the boundary conditions for small normal perturbations ~exp (ikr) describing the problem posed. The magnetization of the liquid varies linearly with the magnetic field intensity $\mathbf{M} = \chi\mathbf{H}$, $\alpha = \alpha^* - \sigma(T - T^*)$. In this case the potentials of the magnetic field perturbations in all three media satisfy Laplace's equation

$$\Delta\Phi_i = 0 \quad (i = 1,\ 2,\ 3), \tag{1}$$

where the subscript 3 refers to the solid nonmagnetic body ($z < -l$), 2 to the upper semiinfinite massif of magnetic liquid ($z > 0$), and 1 to the layer ($-l < z < 0$).

Since we neglect the temperature dependence of the magnetization, the perturbations of the magnetic field are related to the velocity and temperature perturbations only through the boundary conditions at the free surface [1, 2]. Therefore, it is expedient to simplify the boundary-value problem by eliminating the potentials of the magnetic field perturbations Φ_i. By solving (1) and satisfying the boundary condtions we find

$$\Phi_1 = \frac{(M_1 - M_2)\,F\left[\dfrac{1+\mu_1 \operatorname{th} kl}{\mu_1 + \operatorname{th} kl}\operatorname{sh} kz + \operatorname{ch} kz\right]\exp(i\mathbf{kr})}{1 + \dfrac{\mu_1(1+\mu_1 \operatorname{th} kl)}{\mu_2(\mu_1 + \operatorname{th} kl)}}, \tag{2}$$

$$\Phi_2 = C_2 \exp(-kz)\exp(i\mathbf{kr}), \quad \Phi_3 = C_3 \exp(kz)\exp(i\mathbf{kr}).$$

By eliminating the Φ_i, the pressure, and the longitudinal velocity components from the thermomechanical equations for the magnetizable liquid and the boundary conditions, we obtain the dimensionless form of the initial boundary-value problem:

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$$(d^2/dz^2 - k^2)^2 v_1 = 0, \quad (d^2/dz^2 - k^2)^2 v_2 = 0, \tag{3}$$

$$(d^2/dz^2 - k^2)\, T_1 + v_1 = 0, \quad (d^2/dz^2 - k^2)\, T_2 + (\gamma_2 \varkappa_1/\gamma_1 \varkappa_2)\, v_2 = 0, \tag{4}$$

at z = −1

$$v_1 = dv_1/dz = 0, \quad \text{a) } T_1 = 0, \quad \text{b) } dT_1/dz = 0, \tag{5}$$

at z = 0

$$v_1 = v_2 = 0, \quad dv_1/dz = dv_2/dz, \tag{6}$$

$$T_1 - F = T_2 - (\lambda_1/\lambda_2)\, F, \quad dT_1/dz = (\lambda_1/\lambda_2)\, dT_2/dz, \tag{7}$$

$$d^2 v_1/dz^2 - (\eta_2/\eta_1)\, d^2 v_2/dz^2 = -\mathrm{Ma}\, k^2 (T_1 - F), \tag{8}$$

$$\mathrm{C}\, \mathrm{Bo}\, (d^3/dz^3 - 3k^2 d/dz)(v_1 - \eta_2 v_2/\eta_1) = k^2 F (\mathrm{Bo} + k^2 - k\varphi\, \mathrm{Si}\sqrt{\mathrm{Bo}}), \tag{9}$$

$$\gamma_1\lambda_1 = \gamma_2\lambda_2, \quad \varphi \equiv \mu_2/[1 + \mu_2(\mu_1 + \operatorname{th} k)/\mu_1(1 + \mu_1 \operatorname{th} k)],$$

where F is the perturbation of the free surface.

In going to dimensionless quantities the following scales were used: length, layer thickness l; temperature, $\gamma_1 l$; velocity, $\varkappa_1/l$.

The parameters characterizing the problem posed have the following form: the Bond number $\mathrm{Bo} = (\rho_1 - \rho_2) g l^2/\alpha$, the Marangoni number $\mathrm{Ma} = \sigma\gamma_1 l^2/\eta_1\varkappa_1$, the gravitational number $\mathrm{C} = \eta_1\varkappa_1/(\rho_1 - \rho_2) g l^3$, the coupling parameters λ_2/λ_1, η_2/η_1, $\varkappa_1/\varkappa_2$, the relative magnetic permeabilities of the layer and the upper massif μ_1 and μ_2; $\mathrm{Si} = \mu_0(M_1 - M_2)^2/\sqrt{(\rho_1 - \rho_2) g\alpha}$ is the surface instability number of the magnetizable liquid.

The boundary conditions at the free surface were obtained by using the continuity conditions for velocity, temperature, heat flux, tangential component of the magnetic field intensity and the normal component of the magnetic induction, and also the conditions for the normal and tangential stresses.

The boundary conditions for the temperature (5) correspond to the following: a) the temperature is specified on the lower plane (its perturbation vanishes); b) a constant heat flux is specified (the heat flux related to the temperature perturbation vanishes). Henceforth, all expressions denoted by the letters a and b correspond to one of the two forms of conditions for the temperature at the solid boundary (z = −1).

We investigate the stability of equilibrium for monotonic perturbations only.

The solution of problem (3)-(9) can be obtained in the usual way, and leads to the following limits of stability:

$$\text{a)}\quad \mathrm{Ma} = \frac{8k^2\left[\operatorname{sh} k \operatorname{ch} k - k + \dfrac{\eta_2}{\eta_1}(\operatorname{sh}^2 k - k^2)\right]\left(\operatorname{ch} k + \dfrac{\lambda_2}{\lambda_1}\operatorname{sh} k\right)}{\dfrac{8k^4 \mathrm{C}\sqrt{\mathrm{Bo}}\,(\operatorname{sh} k + \operatorname{ch} k)}{\dfrac{\sqrt{\mathrm{Bo}}}{k} + \dfrac{k}{\sqrt{\mathrm{Bo}}} - \mathrm{Si}\,\varphi} + \left[\operatorname{sh}^3 k - k^3 \operatorname{ch} k - \dfrac{\varkappa_1}{\varkappa_2}(\operatorname{sh}^2 k - k^2)\operatorname{sh} k\right]}, \tag{10}$$

$$\text{b)}\quad \mathrm{Ma} = \left\{ 8k^2\left[\operatorname{sh} k \operatorname{ch} k - k + \frac{\eta_2}{\eta_1}(\operatorname{sh}^2 k - k^2)\right]\left(\operatorname{sh} k + \frac{\lambda_2}{\lambda_1}\operatorname{ch} k\right)\right\}\left\{\frac{8k^4 \mathrm{C}\sqrt{\mathrm{Bo}}\,(\operatorname{sh} k + \operatorname{ch} k)}{\dfrac{\sqrt{\mathrm{Bo}}}{k} + \dfrac{k}{\sqrt{\mathrm{Bo}}} - \mathrm{Si}\,\varphi} + \right.$$

$$\left. + \left[\operatorname{sh}^2 k \operatorname{ch} k + k^2 \operatorname{ch} k - 2k \operatorname{sh} k - \frac{\varkappa_1}{\varkappa_2}(\operatorname{sh}^2 k - k^2)\operatorname{ch} k\right]\right\}^{-1}. \tag{11}$$

Equations (10) and (11) show that the convective (thermocapillary) and surface (magnetic) instability mechanisms are interconnected. The nature of this interconnection depends on the deformability of the surface (parameters Bo and C) and the characteristics of the adjoining media (parameters η_2/η_1, λ_2/λ_1, $\varkappa_1/\varkappa_2$, μ_2, and μ_1).

For C = 0 the deformation of the surface does not affect the thermocapillary instability mechanism. In this case the thermocapillary and magnetic mechanisms are not interconnected; i.e., the instability of the surface arises only from the magnetic mechanism, as is the case for an isothermal magnetizable liquid, and does not lead to the initiation of motion; the convective instability arises only from the thermocapillary mechanism and does not lead to deformation of the surface.

For a better understanding of the physical meaning of the interconnection under study we consider the limiting cases of wavelengths which are very short and very long in comparison with the layer thickness, i.e., $k \to \infty$ and $k \to 0$.

1. Large Wave Numbers, $k \to \infty$. In this case Eqs. (10) and (11) lead to the same result – the boundary conditions for the temperature on the undeformable boundary ($z = -1$) do not affect the stability,

$$\mathrm{Ma} = \frac{8k^2(1+\eta_2/\eta_1)(1+\lambda_2/\lambda_1)}{\dfrac{8k^4 C\sqrt{\mathrm{Bo}}}{\operatorname{sh} k\left(\dfrac{k}{\sqrt{\mathrm{Bo}}}+\dfrac{\sqrt{\mathrm{Bo}}}{k}-\dfrac{\mu_1\mu_2\,\mathrm{Si}}{\mu_1+\mu_2}\right)}+1-\dfrac{\varkappa_1}{\varkappa_2}}. \tag{12}$$

If $1-\varkappa_1/\varkappa_2 \neq 0$ and $\mathrm{Si} \neq \dfrac{\mu_1+\mu_2}{\mu_1+\mu_2}\left(\dfrac{\sqrt{\mathrm{Bo}}}{k}+\dfrac{k}{\sqrt{\mathrm{Bo}}}\right)$, Eq. (12) reduces to the following:

$$\mathrm{Ma} = \frac{8k^2(1+\eta_2/\eta_1)(1+\lambda_2/\lambda_1)}{1-\varkappa_1/\varkappa_2}. \tag{13}$$

Equation (13) agrees with the result obtained in [3] for a layer of ordinary liquid; i.e., the deformation of the surface can be neglected for short wavelength perturbations, and the magnetic mechanism of surface instability is interconnected with the thermocapillary mechanism only over a narrow range of Si values near the critical value Si_T for an isothermal liquid. In addition, as $k \to \infty$ the neutral Ma (k) curve for Si values far from Si_T depends on the parameters η_2/η_1, λ_2/λ_1, and $\varkappa_1/\varkappa_2$ in exactly the same way as for an ordinary liquid [3].

2. Small Wave Numbers, $k \to 0$. This approximation leads to different results in the two cases considered:

$$\text{a)}\quad \mathrm{Ma} = \frac{80(1+k\eta_2/2\eta_1)(1+k\lambda_2/\lambda_1)}{k^2-5\varkappa_1/\varkappa_2+120\,C\sqrt{\mathrm{Bo}}/[\sqrt{\mathrm{Bo}}+k^2/\sqrt{\mathrm{Bo}}-k\,\mathrm{Si}\,\mu_2/(1+\mu_2)]}, \tag{14}$$

$$\text{b)}\quad \mathrm{Ma} = \frac{80(1+k\eta_2/2\eta_1)(k+\lambda_2/\lambda_1)}{4k/3-5\varkappa_1/\varkappa_2+120C\sqrt{\mathrm{Bo}}/[\sqrt{\mathrm{Bo}}+k^2/\sqrt{\mathrm{Bo}}-k\,\mathrm{Si}\,\mu_2/(1+\mu_2)]}. \tag{15}$$

Equations (14) and (15) can be minimized analytically if the following conditions are satisfied:

in case a)

$$k \ll \frac{\lambda_1}{\lambda_2},\quad k \ll \frac{2\eta_1}{\eta_2},\quad k^3-5k\frac{\varkappa_1}{\varkappa_2} \ll \frac{120\,C\sqrt{\mathrm{Bo}}}{\dfrac{\sqrt{\mathrm{Bo}}}{k}+\dfrac{k}{\sqrt{\mathrm{Bo}}}-\dfrac{\mathrm{Si}\,\mu_2}{1+\mu_2}},$$

in case b)

$$k \ll \frac{\lambda_2}{\lambda_1},\quad k \ll \frac{2\eta_1}{\eta_2},\quad \frac{4k^2}{3}-5k\frac{\varkappa_1}{\varkappa_2} \ll \frac{120\,C\sqrt{\mathrm{Bo}}}{\dfrac{\sqrt{\mathrm{Bo}}}{k}+\dfrac{k}{\sqrt{\mathrm{Bo}}}-\dfrac{\mathrm{Si}\,\mu_2}{1+\mu_2}}.$$

As a result we obtain the following critical values of the wave number and Marangoni number:

$$\text{a)}\quad \mathrm{Ma}^{\mathrm{cr}} = \frac{2}{\dfrac{3C}{1-\dfrac{\mu_2^2\mathrm{Si}^2}{4(1+\mu_2)^2}}-\dfrac{\varkappa_1}{8\varkappa_2}},\qquad k^{\mathrm{cr}} = \frac{\mathrm{Si}\sqrt{\mathrm{Bo}}\,\mu_2}{2(1+\mu_2)}, \tag{16}$$

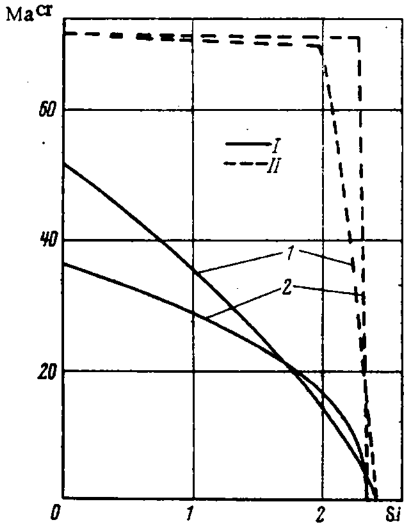

Fig. 1

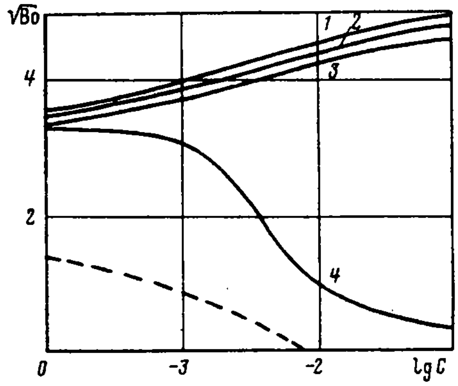

Fig. 2

Fig. 1. Ma^{cr} as a function of Si for $\varkappa_1/\varkappa_2 = \lambda_2/\lambda_1 = 0$, C = 0.01 [I) $\eta_2/\eta_1 = 0$; II) $\eta_2/\eta_1 = 10$]: 1) $\sqrt{Bo} = 1$; 2) $\sqrt{Bo} = 3$.

Fig. 2. Domain of continuity of $k^{cr}(Si)$ curves: 1) $\eta_2/\eta_1 = 0$; 2) 1; 3) 2; 4) 10.

$$\text{b)}\quad Ma^{cr} = \frac{\dfrac{2\lambda_2}{\lambda_1}}{\dfrac{3C}{1-\dfrac{Si^2\mu_2^2}{4(1+\mu_2)^2}} - \dfrac{\varkappa_1}{8\varkappa_2}}, \quad k^{cr} = \frac{Si\sqrt{Bo}\,\mu_2}{2(1+\mu^2)}. \tag{17}$$

It is easy to see that for $\varkappa_1/\varkappa_2 = 0$ and heating from below, the magnetic mechanism of surface instability has a destabilizing effect on the instability of the layer; i.e., as Si is increased, the critical values of Ma decreases to zero for $Si = 2(1 + \mu_2)/\mu_2$.

The interconnection of the thermocapillary and magnetic mechanisms for $\varkappa_1/\varkappa_2 > 0$ is of a somewhat special character, since in this case pure thermocapillary instability is possible for transfer in both directions (heating from above or from below).

3) For arbitrary values of the parameters Eqs. (10) and (11) were analyzed numerically. The parameters μ_1 and μ_2 were fixed at $\mu_1 = 1.5$, $\mu_2 = 2$, and the values of all other parameters were varied. It should be emphasized that everywhere it is assumed that the density of the lower liquid ρ_1 is larger than that of the upper ρ_2, since otherwise mechanical equilibrium could not exist.

If $\varkappa_1/\varkappa_2 = 0$ and the medium is heated from above, instability arises from the magnetic mechanism, and the thermocapillary mechanism has a stabilizing effect. For heating from below, instability of the layer can arise from both the thermocapillary and magnetic mechanisms, and the interconnection between them becomes stronger with increasing values of the gravitational number C. In addition, the layer becomes less stable with an increase in the gravitational number; i.e., the critical values of the Marangoni number are decreased. For Si = 0 the critical values of Ma also decrease with an increase of the Bond number Bo. The nature of the $Ma^{cr}(Si)$ dependence on the parameters Bo, C, η_2/η_1, and λ_2/λ_1 is more complicated, and will be explained in the analysis of the graphs. The dependences of the critical value of the Marangoni number on the surface instability number Si for various values of Bo and η_2/η_1 are shown in Fig. 1. All the graphs presented in this paper are for an isothermal lower boundary; i.e., they are calculated from Eq. (10). In order to save space the results for a thermally insulated boundary are not shown graphically. It is clear from Fig. 1 that the critical values of the Marangoni number increase in the ratio η_2/η_1; the nature of the $Ma^{cr}(Si)$ dependence is changed also: the solid curves ($\eta_2/\eta_1 = 0$) are smooth, but the open curves ($\eta_2/\eta_1 = 10$) have a sharp bend. Accordingly, the curves for $k^{cr}(Si)$ are continuous or have a discontinuity. In Fig. 2 the ranges of values of the parameters Bo and C for which the $k^{cr}(Si)$ curves are continuous lie between the open curve and the solid curves 1-4. For values of C and Bo which lie outside the curves the $k^{cr}(Si)$ curves have a discontinuity.

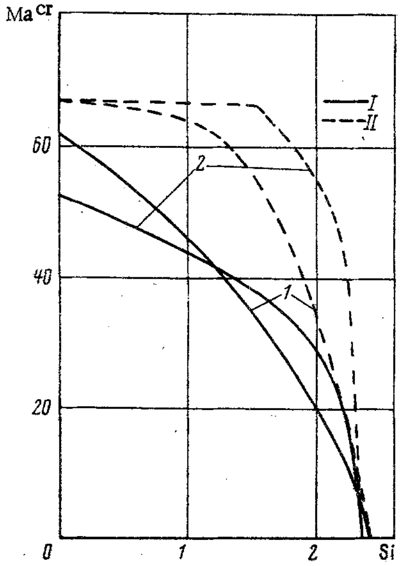

Fig. 3

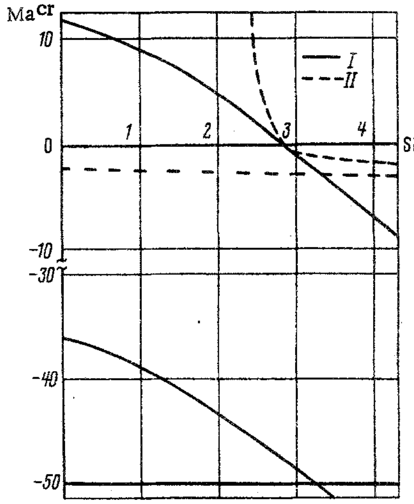

Fig. 4

Fig. 3. Critical $Ma^{cr}(Si)$ curves for $\varkappa_1/\varkappa_2 = 0$, $\eta_2/\eta_1 = 1$ [I) $\lambda_2/\lambda_1 = 0$; II) $\lambda_2/\lambda_1 = 1$]; 1) $\sqrt{Bo} = 1$; 2) 3.

Fig. 4. Ma^{cr} as a function of Si for $C = 0.1$, $\sqrt{Bo} = 0.1$: I) $\varkappa_1/\varkappa_2 = 1$; II) $\varkappa_1/\varkappa_2 = 10$.

An analysis of these graphs shows that as η_2/η_1 is increased the domain of continuity of the $k^{cr}(Si)$ curves (smoothness of the $Ma^{cr}(Si)$ curves) is narrowed, i.e., the interconnection between the thermocapillary and magnetic mechanisms is weakened.

The effect of a change of λ_2/λ_1 on the stability of the layer was investigated also. To do this we considered the results of a numerical analysis of Eq. (10) for fixed $\varkappa_1/\varkappa_2$, η_2/η_1, and C for various values of Bo and λ_2/λ_1. It was found that as λ_2/λ_1 increases, the critical value of the Marangoni number increases. The limiting case $\lambda_2/\lambda_1 \to \infty$ indicates a transition to an isothermal surface when the gradient of the surface tension vanishes at the boundary, and consequently thermocapillary instability is impossible; Ma^{cr} approaches infinity. In this limiting case the instability of the layer will be produced by the magnetic mechanism of surface instability. The critical $Ma^{cr}(Si)$ curves are shown in Fig. 3 for various values of Bo and λ_2/λ_1.

So far in the numerical analysis of Eqs. (10) and (11), we have limited ourselves to the case when the thermal diffusivity of the lower fluid can be neglected in comparison with that of the upper ($\varkappa_1/\varkappa_2 = 0$). In such situations the thermocapillary instability is related to convection in the lower phase only; the upper liquid is passive. If the thermal diffusivity of the lower phase is not negligible in comparison with that of the upper ($\varkappa_1/\varkappa_2 \neq 0$), the thermocapillary instability is related to convection in both lower and upper phases. Therefore instability is possible even if there is no magnetic field for heating both from below and above. It is quite natural that in the case analyzed ($\varkappa_1/\varkappa_2 \neq 0$) the deformability of the surface, characterized by the parameter C, will have a significant effect on the stability; with its increase the limiting values of the temperature gradients are decreased for heating from below and increased for heating from above. In a magnetic field the surface of a magnetizable liquid becomes less stable, i.e., more deformed, and therefore the dependence of the critical values of the Marangoni number on Si will be similar to the dependence of Ma^{cr} on C, the only difference being that for heating from below Ma^{cr} approaches zero asymptotically as C increases, while Ma^{cr} decreases considerably more sharply with increasing Si and becomes zero for $Si = Si_T$. This is related to the presence of a magnetic mechanism of surface instability in a magnetizable liquid. For heating from above the critical values of Ma increase with increasing C and increasing Si. These characteristics are illustrated by the $Ma^{cr}(Si)$ curves shown in Fig. 4. It is easy to see that the domain of stability is bounded from above and from below.

It should be noted that when the heat flux vanishes on the lower boundary, the layer under investigation will be more unstable than in the case analyzed; nevertheless, the qualitative relations established above will hold.

NOTATION

x, y, z	are the Cartesian coordinates;
T, v	are the perturbations of temperature and z component of velocity;
Φ	is the potential of magnetic field perturbations;
M	is the magnetization of liquid;
H	is the magnetic field intensity;
μ_0	is the magnetic permeability of vacuum;
χ	is the magnetic susceptibility;
η	is the dynamic viscosity;
λ	is the thermal conductivity;
$\varkappa$	is the thermal diffusivity;
g	is the acceleration due to gravity;
β	is the volume coefficient of expansion;
K	is the pyromagnetic coefficient;
l	is the layer thickness;
γ	is the temperature gradient;
$\mathbf{k} = [k_x, k_y, 0]$	is the wave vector;
α	is the surface tension;
$\sigma = -(\partial\alpha/\partial T)/$	
α^*; μ	is the relative magnetic permeability;
F	is the amplitude of perturbations of free surface.

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CONVECTIVE MOTION OF A CONDUCTING LIQUID IN AN ELECTROMAGNETIC FIELD, TAKING INTO ACCOUNT FINITE WALL THICKNESS AND THERMAL CONDUCTIVITY

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The effect of the temperature-dependent electrical conductivity of the liquid and the finite wall thickness and the thermal conductivity on stability is investigated in a linear formulation.

In [1] the convective instability of a liquid layer in a magnetic field was investigated, taking into account the finite wall thickness and thermal conductivity. In the present work, stability of this type is investigated taking account of the temperature dependence of the electrical conductivity.

1. Formulation of the Problem

Consider an infinite horizontal layer of electrically conducting liquid of thickness B, the electrical conductivity of which depends linearly on the temperature $\sigma=\sigma_{00}\,[1+\alpha(T-T_{00})]$ under the condition that $|\alpha(T-T_{00})| \ll 1$ [2]. The walls bounding the layer have the same finite thickness and thermal conductivity λ_1. The temperatures at the external surfaces of the walls are given to be constant, but different (T_1 is the temperature at the lower wall and T_2 at the upper wall). In the y direction, a constant external electric field

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